Duke Quantum Center

Optimal decoding of 2D compass codes under coherent noise

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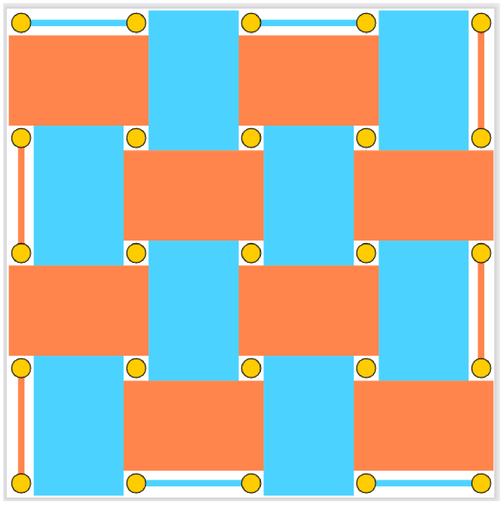
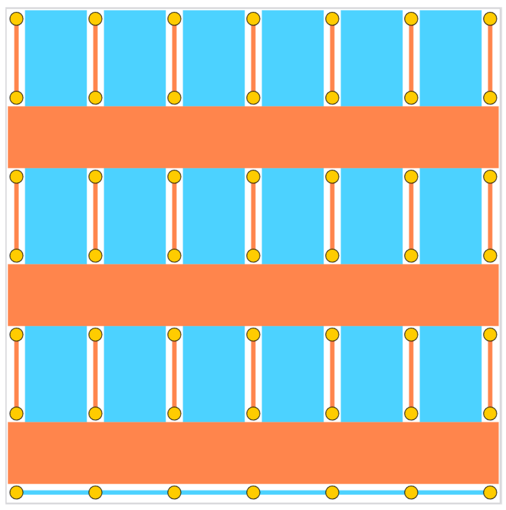
2D COMPASS CODES [5,6]



The 7x7 rotated surface code

The leading proposal for QEC, no analytical constant lower bound on the threshold under coherent noise.



The 7x7 $\mathcal{C}_{\mathcal{L},2}$ Z-stacked Shor code

A very similar compass code that has efficiently calculable logical channel under coherent noise [8].

COHERENT NOISE

Coherent noise stems from unitary over- and under-rotation on qubits. The worst case in terms of logical coherence is when each qubit experiences the same rotation. The main issue with coherent noise is that repeated application increases the average infidelity **quadratically** compared to incoherent noise where average infidelity only grows **linearly**.

$$U_Z = (e^{-i\frac{\theta}{2}Z})^{\otimes n}$$

LOGICAL CHANNEL UNDER COHERENT NOISE

Logical channel

1. Apply U_Z
2. Measure the X-syndrome s
3. Decode the syndrome and apply recovery operator C_s

For codes with single logical qubit, even stabilizers and odd distance it's a **logical rotation [3,4]**

$$|\psi_L\rangle \to \cos\left(\frac{\theta_s}{2}\right)|\psi_L\rangle - i\sin\left(\frac{\theta_s}{2}\right)Z_L|\psi_L\rangle$$

The average logical infidelity is a good performance metric [7], and is defined as a weighted sum over the logical angles across all syndromes [4]:

$$r_1 = \sum_s p_s(1-\cos(\theta_s))$$

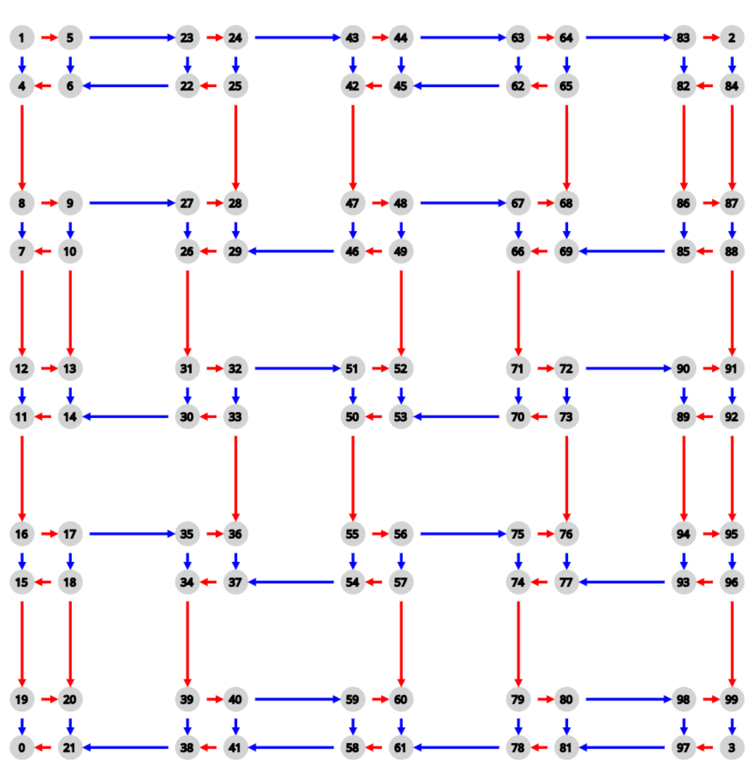
OPTIMAL DECODING

Only two options for recovery: C_s and $\bar{Z}\,C_s$

$$|\psi_L\rangle \to \cos\left(\frac{\theta_s}{2}\right)|\psi_L\rangle - i\sin\left(\frac{\theta_s}{2}\right)Z_L|\psi_L\rangle \quad + \quad \arctan(x) - \arctan(x^{-1}) = \frac{\pi}{2}$$

$$\theta_{C_s} = \pi - \theta_{\bar{Z}C_s}$$

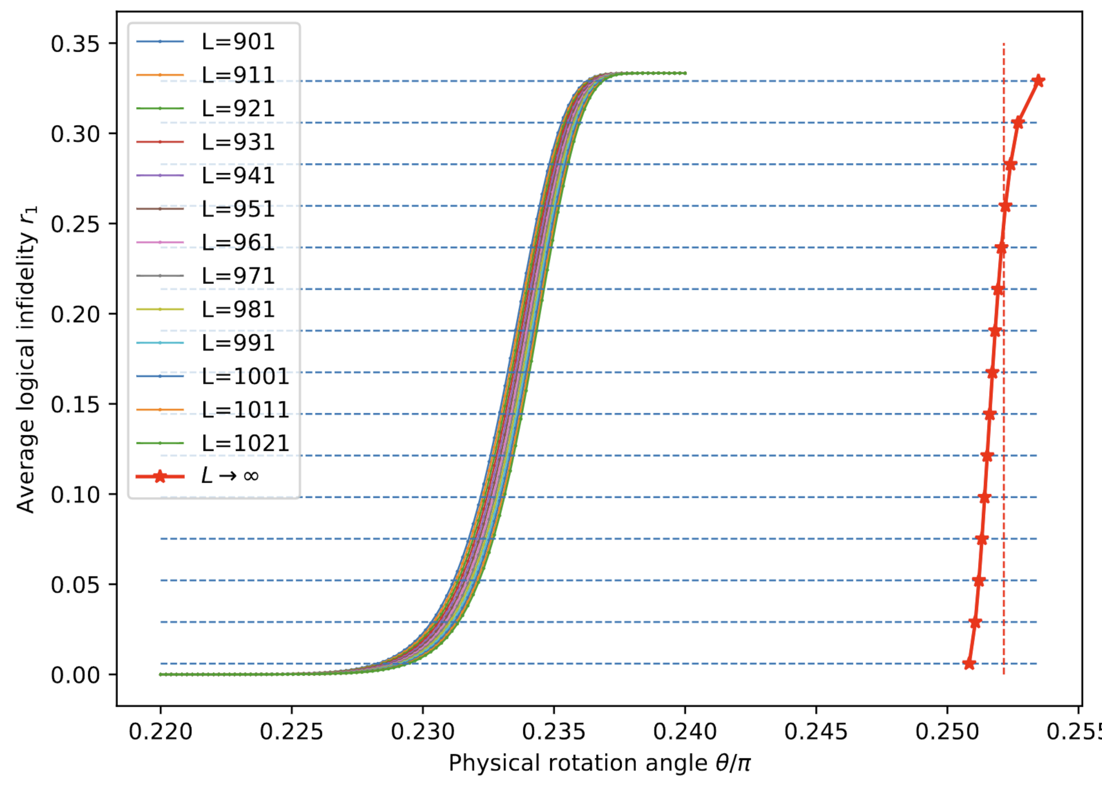
From the MWPM angle, we can calculate the optimal angle!



Majorana mode simulator [3,9] features:

- sample syndromes $\mathcal{O}(n^2)$ **complexity**
- calculate the logical angle in $\mathcal{O}(n^2)$ **complexity** for given syndrome

FINITE SIZE SCALING OF Z-STACKED SHOR CODES



Under optimal decoding infidelity curves behave differently than under MWPM decoding. There is **no clear intersection point**, and the sigmoids keep shifting. We need to use **finite size scaling (FSS) methods** to estimate the infinite size behavior.

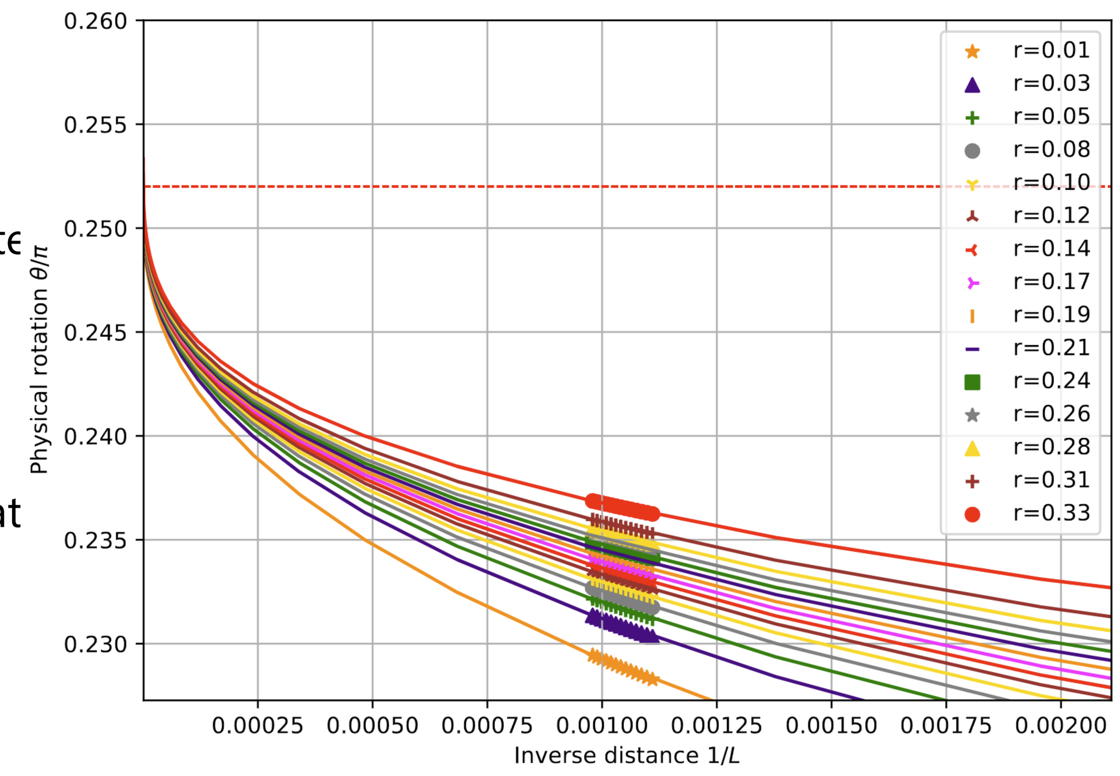
We found that the ansatz

$$\theta_{th} + b(1/L)^c$$

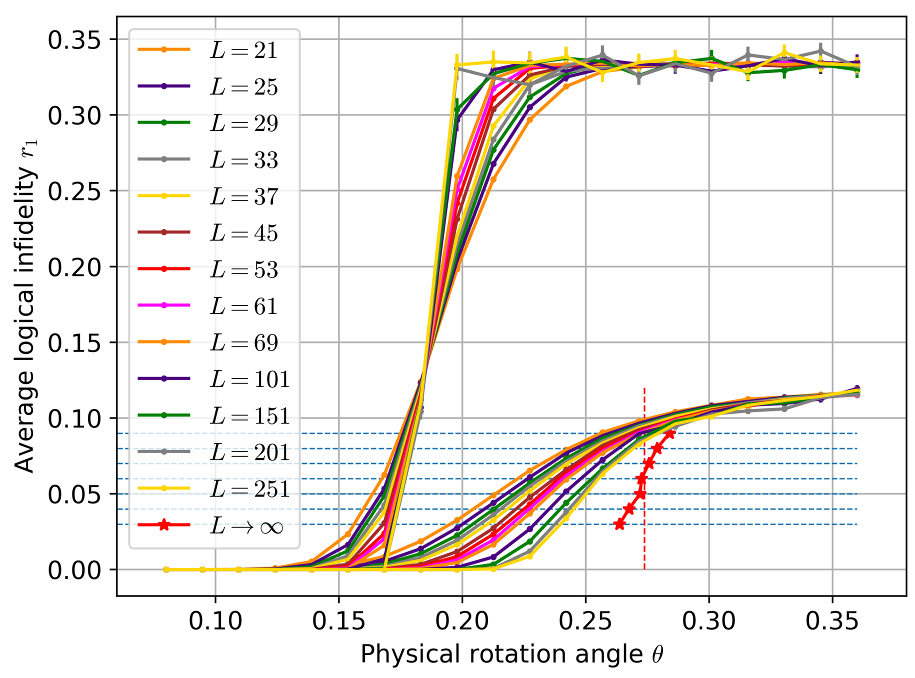
is a high quality fit with R^2 > 0.999. The offset is an estimate of the threshold for $\mathcal{C}_{\mathcal{L},2}$

$$(0.252 \pm 0.001)\pi$$

slightly higher than previous result $\frac{\pi}{4}$ [8]. **Reason**: FSS error – reduces at higher distances, increases when lower distances are included.



ROTATED SURFACE CODE



Similarly, under optimal decoding infidelity curves behave differently than under MWPM decoding. There is **no clear intersection point**, and the sigmoids keep shifting. We need to use **finite size scaling methods** to estimate the infinite size behavior. Under MWPM, the usual intersection behavior identifies the previously found ~ 0.19 π

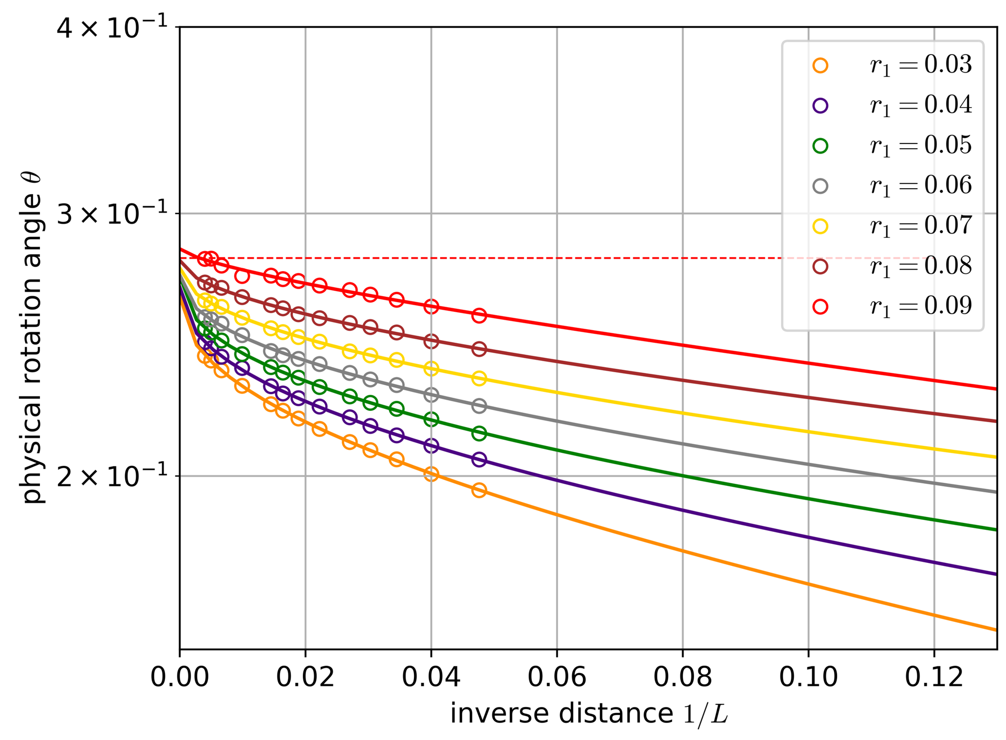
Using the same ansatz The offset is an estimate of the threshold for the RSC:

$$(0.274 \pm 0.010)\pi$$

slightly lower than previous results

$$(0.28 \pm 0.01)\pi$$

using minimum infidelity [2]. As Z-stacked Shor results suggests that FSS might overestimate the threshold, thus it might be even lower.



DISCUSSION AND FUTURE WORK

- Optimal decoding + finite size scaling slightly deviates from previous predictions, due to FSS errors – exact characterization of errors is subject of future work.
- Exploring the family of compass codes under optimal decoding is future work extending our previous results with MWPM data only
- Logical gate design might also benefit from optimal decoding

REFERENCES

[1] E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, "Topological quantum memory," Journal of Mathematical Physics, vol. 43, no. 9, 2002, pp. 4452–4505.
[2] F. Venn, J. Behrends, and B. Béri, "Coherent-error threshold for surface codes from Majorana delocalization," Phys. Rev. Lett., vol. 131, no. 6, 2023, p. 060603.
[3] S. Bravyi, M. Englbrecht, R. König, and N. Peard, "Correcting coherent errors with surface codes," npj Quantum Inf, vol. 4, no. 1, 2018, p. 55.
[4] J. K. Iverson and J. Preskill, "Coherence in logical quantum channels," New J. Phys., vol. 22, no. 7, 2020, p. 073066.
[5] D. Bacon, "Operator Quantum Error Correcting Subsystems for Self-Correcting Quantum Memories," Phys. Rev. A, vol. 73, no. 1, 2006, p. 012340.
[6] M. Li, D. Miller, M. Newman, Y. Wu, and K. R. Brown, "2D Compass Codes," Phys. Rev. X, vol. 9, no. 2, 2019, p. 021041.
[7] S. J. Beale, J. J. Wallman, M. Gutiérrez, K. R. Brown, and R. Laflamme, "Quantum Error Correction Decoheres Noise," Phys. Rev. Lett., vol. 121, no. 19, 2018, p. 190501.
[8] B. Pato, J. W. Staples Jr., and K. R. Brown, "Logical coherence in 2D compass codes," arXiv preprint: arXiv.2405.09287, 2024.
[9] B. Pato, J. W. Staples Jr., "msim, a Majorana mode simulator,"https://gitlab.com/duke-artiq/error-correction/msim

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